

# SOME RESULTS IN SUPERSONIC COMBUSTION\*

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## ABSTRACT

Some basic aspects of supersonic combustion are discussed, concerning the use of air-breathing engines for the propulsion of space vehicles within the atmosphere. The advantages and drawbacks of using diffusion flames for supersonic combustion are discussed. An analysis of the supersonic turbulent mixing process is included. The influence on the flow field of the heat addition due to the diffusion flame is also analyzed. Finally, chemical kinetic non-equilibrium effects with special emphasis on flame extinction are treated by considering, first, the laminar diffusion case and, afterwards, a tentative extension to the turbulent case.

## INTRODUCTION

In recent years considerable attention has been given to the study of supersonic combustion problems, in order to develop a hypersonic vehicle that obtains the necessary oxidizer by air breathing [1].

Although present and planned chemical rocket programs will provide the propulsion requirements of the near future, the costs of developing these chemical rocket systems are enormous.

It appears that hypersonic air breathing would make it possible to place heavy loads in orbit, by powering some stage with this form of propulsion—using lighter, smaller, more maneuverable vehicles than would be feasible with rocket propulsion. Furthermore, sentpre results show that direct operating costs for ramjet powered hypersonic airplanes would be no greater than for the supersonic transport.

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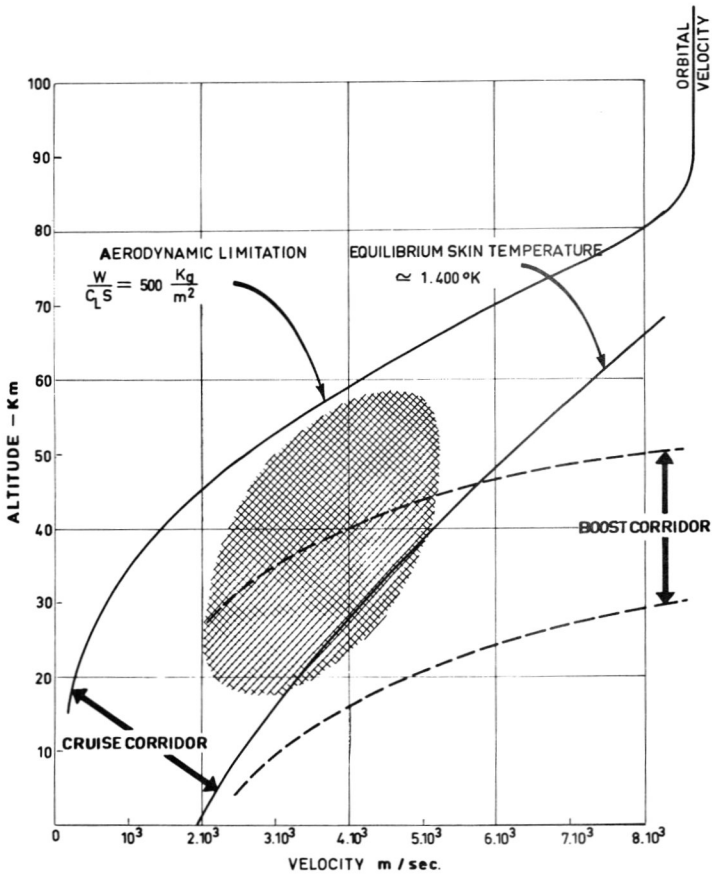


Figure 1.

The concept of the "flight corridor" (Fig. 1) is often used to illustrate the boundaries of hypersonic vehicle operation [2,3]. The upper limit for cruise is obtained when the maximum lift (plus centrifugal force) is equal to the weight. The lower limit is determined when the equilibrium skin temperature reaches a certain value.

In a boost vehicle high accelerations are required so as to increase rapidly the kinetic energy of the vehicle at low altitudes, and thus maximize the payload. Therefore, a relatively low corridor is obtained. Of course, such a decrease in altitude will result in higher, highly variable, heat-transfer rates, and the short-term transient nature of heating is important. In Fig. 1 the shaded zone represents the region covered in the numerical applications presented in this paper.

Two fundamentally different modes of combustion in a supersonic stream have been considered up to now: diffusion flame and premixed detonative combustion.

Dr. A. Ferri suggested, in the First ICAS Congress in Madrid [4], the use of a diffusion flame for supersonic combustion. In Ferri's design [5] the fuel is injected at the inlet parallel to the air flow. Mixing, mainly turbulent mixing, begins immediately, and combustion quickly follows.

In the detonation mode, studied by Gross and Chinitz [6], Nicholls [7], Rhodes [8], and others, the oxidizer and fuel are completely premixed upstream of the combustor, and chemical reaction is initiated by aerodynamic waves which increase the pressure and temperature of the mixture.

It seems that this latter mode of combustion presents certain operative drawbacks primarily because of the interdependence of combustor geometry and of aerodynamic and heat-release effects. Furthermore, the energy dissipation associated with the detonation wave is rather severe, and, as we shall show below, the perturbation that the heat release produces in the aerodynamic field is reduced when the heat sources are smoothly distributed along the flow field.

On the other hand, in the diffusive mode, the fact that combustion is controlled by mixing eliminates the need for variable inlet geometry.

However, for the diffusion flame to exist the chemical reaction time must be small compared with the mixing or mechanical time. This fact limits the applicability of the diffusive mode of combustion to some regions of the flight corridor.

In the following, work carried out at INTA on three basic problems related to supersonic combustion will be discussed: turbulent mixing, aerodynamic effects of heat release, and nonequilibrium effects in diffusion flames.

## **TURBULENT MIXING PROBLEM**

In the Introduction, the advantages of diffusive burning have been considered.

The most obvious configuration to achieve this mode of combustion seems to be one or several low-velocity jets of fuel, surrounded by a high-speed stream of coflowing oxidant. Mixing and combustion take place under turbulent conditions.

A great deal of theoretical and experimental work has been done recently on the turbulent mixing of two dissimilar gases for both subsonic and supersonic flow, but the corresponding results for the case of combustion

are rather scarce, and much work seems to be needed before a complete understanding of the problem can be achieved.

In the theoretical study of the behavior of a reactive gas injected into an air stream, the gas composition is usually calculated by assuming either frozen or chemical equilibrium conditions. In what follows, we assume equilibrium composition, and by analyzing the internal structure of the diffusion flame we shall determine whether chemical equilibrium prevails or not.

In a diffusion flame, the reaction zone separates the two reacting species which diffuse, through inert gases and combustion products, reaching the flame in stoichiometric proportion.

Since the reacting species burn very rapidly as they reach the flame, the combustion velocity is conditioned to the facility of the species to diffuse across the inert gases and combustion products.

Consequently, we can arrive at a description of some of the most important features of diffusion flames by using the Burke-Schumann assumption of infinitely fast reaction rates [9, 10, 11]. Fuel and oxidizer cannot then coexist, and the zone of combustion becomes very thin. The mixing process is the only factor responsible for the rate of burning, flame location and size. This problem will be treated with more detail below.

The Burke-Schumann assumption eliminates chemical kinetics from the process, reducing the problem to a diffusion one, but the criterion for the extinction of the flame, or for the validity of the assumption, requires substantial additional analysis.

## TURBULENT GAS JETS

By using the aforementioned simplifying assumption, the problem of turbulent mixing and combustion reduces to a mixing problem, with the additional conditions that the fuel and oxidizer do not coexist in any zone of the flow field and reach the flame surface in stoichiometric proportion.

A complete study of the mixing in turbulent gas jets is presented in Ref. 12. Since the results given in this book appear in explicit, simple mathematical form, and are useful for design purposes, we follow in our discussion the same approach.

When a gas jet discharges in a coflowing stream, three different regions appear. There is an "initial region," with a boundary layer separating an inner, inviscid, uniform fuel core from the outer uniform stream of air. This boundary layer thickens by entraining fuel and air from the uniform streams. When this boundary layer reaches the jet axis, a "transition region" begins until a "main region" appears where similarity exists between velocity profiles (Fig. 2).

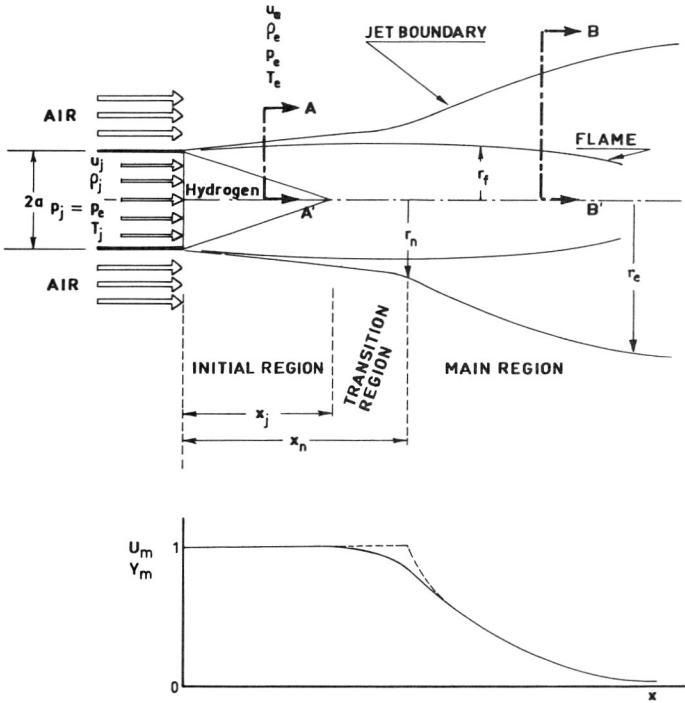


Figure 2.

VELOCITY, TEMPERATURE AND CONCENTRATION PROFILES

According to Taylor’s theory of free turbulence, thermal and mass turbulent diffusivities are identical, and velocity and temperature or concentration profiles are related by the following expression:

$$\frac{U}{U_m} = \left( \frac{T - T_e}{T_m - T_e} \right)^2 = \left( \frac{Y_i}{Y_{im}} \right)^2 \tag{1}$$

This ceases to be true when chemical reactions occur during the mixing process. Then, the relations, Eq. (1), remain valid if the temperature is substituted by the total (including chemical) enthalpy, and the mass fraction  $Y_i$  of a species  $i$  is substituted by a “reduced mass fraction”  $Y$ . (See nomenclature at the end of the paper.)

The experimental velocity profiles of a wide variety of jets (nonisothermal jet in an external stream, supersonic submerged and cooled jets, supersonic jet under off-design discharge conditions) may be very well

correlated with the following formula (“law of  $\frac{3}{2}$ ” or “Schlichting formula”):

$$\frac{u - u_j}{u_e - u_j} = (1 - \eta^{3/2})^2; \quad \eta = \frac{r_e - y}{b} \quad (2)$$

for the initial and transition regions of the jet, and

$$\frac{u - u_e}{u_m - u_e} = (1 - \xi^{3/2})^2; \quad \xi = \frac{y}{r_e} \quad (3)$$

for the main region.

### JET PROPAGATION EQUATION

The propagation equation for a jet of incompressible fluid, derived on the basis of the Prandtl hypothesis concerning the mechanism of the turbulent expansion of a jet, may be expressed by the relation:

$$\frac{dr}{dx} = \pm c \frac{u_m - u_e}{2\bar{u}} \quad (4)$$

where the minus sign is chosen when  $u_e > u_m$ .

This equation is valid for both plane and axially symmetric jets and, with minor changes, is valid for the whole jet.

When studying jets of incompressible fluids, the mean arithmetical value of the axis and outer velocities was chosen as the characteristic velocity  $\bar{u}$ . However, when the densities in the jet and surrounding medium are not identical, the characteristics velocity should be calculated according to the formula

$$\bar{u} = \frac{\rho_j u_j + \rho_e u_e}{\rho_j + \rho_e} \quad (5a)$$

for the initial region, and

$$\bar{u} = \frac{\rho_m u_m + \rho_e u_e}{\rho_m + \rho_e} \quad (5b)$$

for the main region. Or, what is more rigorous

$$\bar{u} = \frac{\int_0^{r_e} \rho u y dy}{\int_0^{r_e} \rho y dy} \quad (6)$$

As is shown by Abramovich, the values of  $dr/dx$  obtained by using Eqs. (5) and (6) are equivalent and agree with experimental results, at least in the case of a submerged jet ( $m = 0$ ), for values of the density ratio in the range  $0 < \rho_j/\rho_e \leq 2.0$ .

In our particular case, when we are concerned with the combustion of hydrogen in air,  $\rho_j/\rho_e \simeq 0.1$ , and Eq. (6) may be used.

Concerning the values of  $c$ , Abramovich points out that, when using Eq. (5), the values are:

For the initial region,  $c = 0.27$ .

For the main region of the jet,  $c = 0.22$ .

In some particular cases, which are of interest, the relations of Eq. (5) may be simplified even more.

For example, when  $\rho_j \ll \rho_e$ ,  $u_j \ll u_e$ , which is the case when hydrogen is mixed with air, and a high initial velocity ratio is used, with the aim of increasing the mixing rate, we have the value:

$$\bar{u} \simeq u_e \simeq u_e + u_j$$

or twice that of the incompressible case. On the other hand, if  $\rho_j \ll \rho_e$ , but  $\rho_j u_j \ll \rho_e u_e$ , and

$$\bar{u} \simeq \frac{\rho_e}{\rho_j} u_e \simeq \frac{\rho_e}{\rho_j} (u_e + u_j)$$

the incompressible value is again obtained, but a factor  $2\rho_j/\rho_e$  must be used to correct the value of  $c$ .

In Fig. 3,  $b/x$ , that is, the increase in thickness of the mixing zone of the jet with downstream distance, is plotted as a function of the velocity ratio  $m$ . The shaded zone corresponds to the values computed, using Eqs. (4) and (6), for a jet of hydrogen burning (assuming chemical equilibrium) in air. We consider different flight conditions corresponding to the shaded zone of Fig. 1, and for each of them, two different temperatures in the combustor entrance, namely 800°K and 1000°K (assuming isentropic compression). The incompressible case is also indicated in the figure.

#### VELOCITY AND CONCENTRATION PROFILES ALONG THE AXIS OF THE JET MAIN REGION

Now, by means of the equation of conservation of momentum defect, we shall establish the law of variation of the velocity along the axis of the jet.

For an isobaric jet this equation is

$$\int_0^{r_e} 2\pi y \rho u (u - u_e) dy = \pi a^2 \rho_j u_j (u_j - u_e) \quad (7)$$

from which we obtain:

$$U_m^2 A_2 (1 - m) + U_m A_1 m = \left(\frac{a}{r_e}\right)^2 \tag{8}$$

where

$$A_1 = \int_0^1 \frac{\rho}{\rho_j} \frac{U}{U_m} 2 \frac{y}{r_e} d\left(\frac{y}{r_e}\right)$$

$$A_2 = \int_0^1 \frac{\rho}{\rho_j} \left(\frac{U}{U_m}\right)^2 2 \frac{y}{r_e} d\left(\frac{y}{r_e}\right)$$

In the incompressible case, taking into account Eq. (3), we obtain:

$$A_{1i} = 0.258, A_{2i} = 0.134$$

The values corresponding to the compressible case are estimated introducing a mean ( $U_y$  weighted) density  $\rho_M$  in each section of the jet, such that

$$\rho_M \int_0^1 \frac{U}{U_m} 2 \frac{y}{r_e} d\left(\frac{y}{r_e}\right) = \int_0^1 \rho \frac{U}{U_m} 2 \frac{y}{r_e} d\left(\frac{y}{r_e}\right)$$

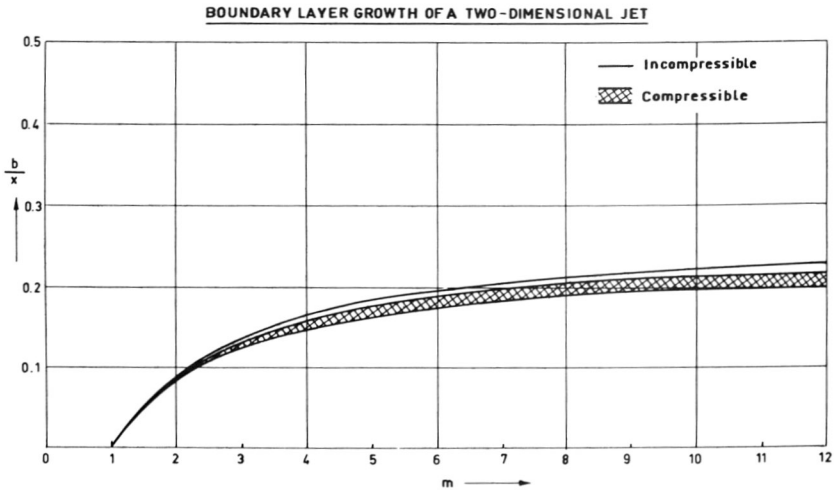


Figure 3.



so that we have

$$A_{1c} = \frac{\rho_M}{\rho_j} A_{1i}; \quad A_{2c} \simeq \frac{\rho_M}{\rho_j} A_{2i}$$

$\rho_M$  is a function of  $x$ , which approaches  $\rho_e$  at sections not too close to the injector exit.

The variation of the "reduced mass fraction,"  $Y$ , may be obtained by writing the corresponding conservation equation

$$\int_0^{r_e} 2\pi y \rho u Y dy = \pi a^2 \rho_j u_j \tag{9}$$

or

$$Y_m U_m B_2 (1 - m) + Y_m B_1 m = \left(\frac{a}{r_e}\right)^2 \tag{10}$$

where

$$B_1 = \int_0^1 \frac{\rho}{\rho_j} \frac{Y}{Y_m} 2 \frac{y}{r_e} d\left(\frac{y}{r_e}\right)$$

$$B_2 = \int_0^1 \frac{\rho}{\rho_j} \frac{U}{U_m} \frac{Y}{Y_m} 2 \frac{y}{r_e} d\left(\frac{y}{r_e}\right)$$

in the incompressible case  $B_{1i} = 0.428$ ,  $B_{2i} = 0.180$ . For the compressible case we use these same values times the factor  $(\rho_M/\rho_j)$ .

### JET BOUNDARIES

Integrating the jet propagation equation, we obtain the following equation:

$$\pm 0.22 \left(\frac{x}{a} - \frac{x_n}{a}\right) = \frac{r_e}{a} - \frac{r_n}{a} + \frac{0.69}{q^2} \left[ \left\langle \left(\frac{r_e}{a}\right)^2 + q^2 \right\rangle^{3/2} - \left\langle \left(\frac{r_n}{a}\right)^2 + q^2 \right\rangle^{3/2} + \left(\frac{r_e}{a}\right)^3 - \left(\frac{r_n}{a}\right)^3 \right] \tag{11}$$

in which

$$q^2 = \frac{4A_2}{A_1^2} \frac{1 - m}{m}$$

and  $x_n$  and  $r_n$  are plotted in Fig. 4, as a function of  $m$ , for the incompressible case.

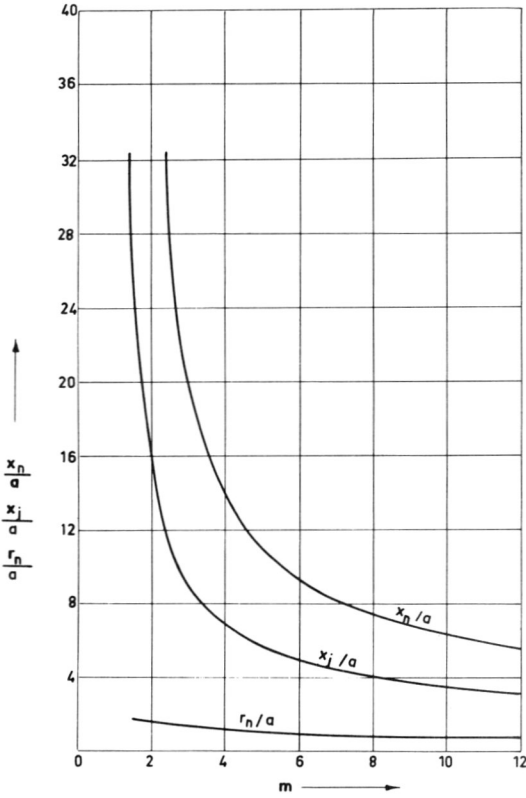


Figure 4.

Reference is made to Chapter V of Ref. 12 for the details of the computation.

From the above we may infer that, in the compressible case,  $Y_m$  and  $U_m$  take the same values as in the incompressible one, but at a section

$$(x/a)_c = (\rho_j/\rho_M)^{1/2} (x/a)_i$$

while the width of the jet is

$$(r_e/a)_c = (\rho_j/\rho_M)^{1/2} (r_e/a)_i$$

Attention is called to the fact that these rules are only valid in the case  $\rho_j \ll \rho_e$ ;  $u_j \ll u_e$ .

In the study of the characteristics of a supersonic combustion diffusive burner, a variable of paramount importance is the length of the combustion

chamber which, as we shall see, is of the order of 100 times the initial radius of the jet.

For such distances, the following approximate relations may be used —

(a) For the dimensionless velocity profile along the jet axis

$$U_m = \frac{1}{mA_{1_i}} \frac{\rho_j}{\rho_e} \left( \frac{a}{r_e} \right)_c^2 \quad (8a)$$

(b) For the reduced concentration

$$Y_m = \frac{\nu Y_{H_2m} + Y_{O_2e}}{\nu + Y_{O_2e}} = \frac{1}{mB_{1_i}} \frac{\rho_j}{\rho_e} \left( \frac{a}{r_e} \right)_c^2 \quad (10a)$$

(c) For the jet propagation equation

$$\left( \frac{x}{a} \right)_c = 0.8 \frac{m^2}{m-1} \frac{\rho_e}{\rho_j} \left( \frac{r_e}{a} \right)_c^3 \quad (11a)$$

Substituting in Eq. (10a) the expression for  $\left( \frac{a}{r_e} \right)_c$  according to Eq. (11a), we have

$$Y_m = 2.0 \frac{m^{1/3}}{(m-1)^{2/3}} \left( \frac{\rho_j}{\rho_e} \right)^{1/3} \left( \frac{x}{a} \right)_c^{-2/3} \quad (12)$$

Equation (12) will be useful in the computation of combustion chamber length.

## FLAME POSITION

Under the assumption that chemical equilibrium is reached from the beginning of the mixing zone, the flame is located at points in which  $Y_{H_2} = Y_{O_2} = 0$ , that is, where

$$Y = Y_f = \frac{Y_{O_2e}}{\nu + Y_{O_2e}} \quad (13)$$

Figure 5 gives the flame shape in four different cases, assuming incompressible flow and introducing the compressibility corrections mentioned before, which are valid only for a certain distance from the injector.

Figure 6 gives the shape of flame and jet under the same limitations, for  $m = 2.5$ .

Finally, Fig. 7 gives the combustion chamber length  $x_f$  as a function of the velocity ratio  $m$ . This has been obtained by using Eq. (12) with  $Y_m = Y_f$ .

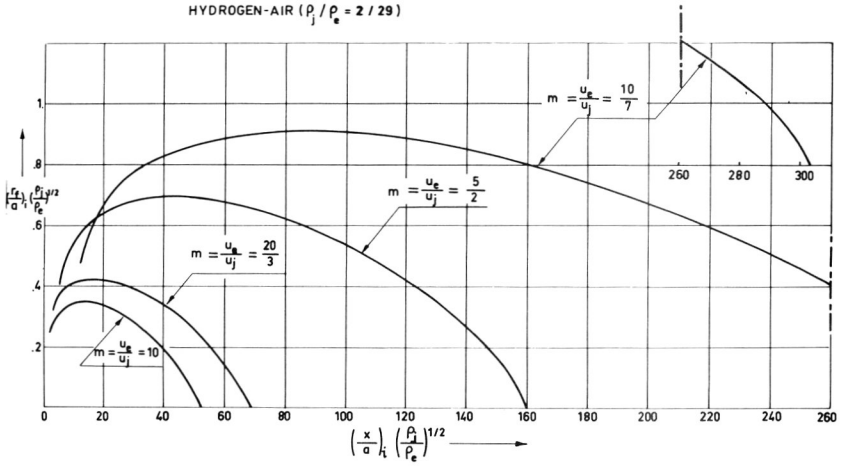


Figure 5.

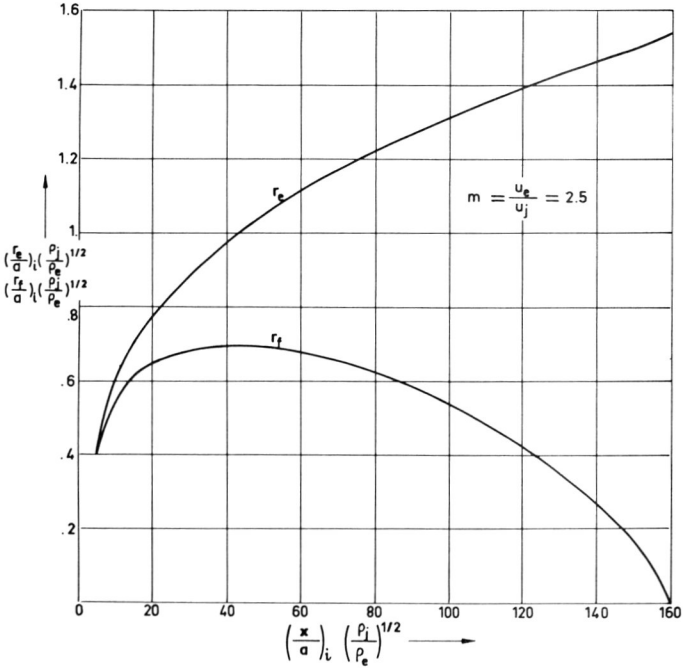


Figure 6.

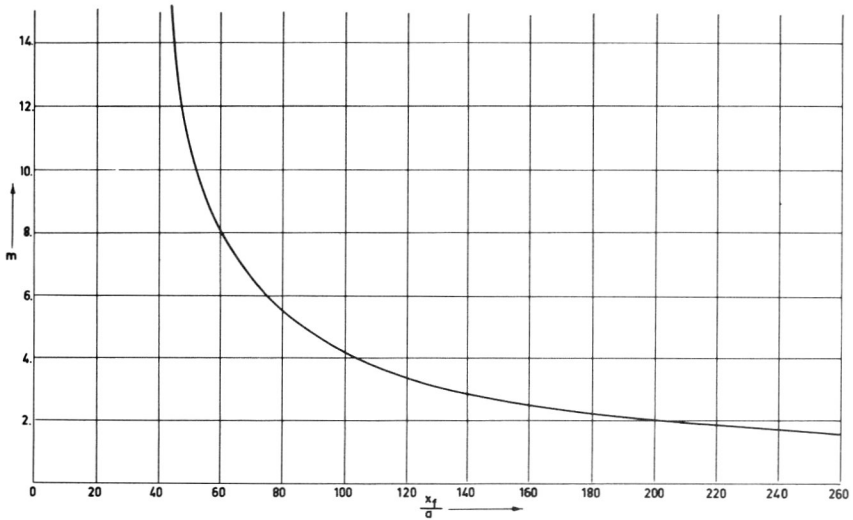


Figure 7.

For large values of  $m$ ,  $x_f$  may be written as:

$$\frac{x_f}{R} = \frac{2^{3/2}}{Y_f^{3/2}} \left( \frac{\rho_j u_j a^2}{\rho_e u_e R^2} \right)^{1/2} \quad (14)$$

where  $R$  is the radius of the combustion chamber.

The factor in parenthesis is approximately equal to the fuel-air ratio. For hydrogen burning in air

$$\frac{x_f}{R} = 580 \left( \frac{\rho_j u_j a^2}{\rho_e u_e R^2} \right)^{1/2} \quad (15)$$

From the foregoing equations, it is shown that when using  $n$  injectors, the  $x_f$  is reduced by a factor of  $n^{-1/2}$ .

It should be noted that, in the foregoing, interference effects have been neglected.

#### HEAT RELEASED BY THE FLAME

In order to estimate the aerodynamic effects of heat release, we must first determine the heat release rate  $Q'$  per unit length along the jet axis.

If  $q$  is the heat release per unit mass of hydrogen,

$$Q' = -q \frac{d}{dx} \int_c^{r_f} \rho u Y_{H_2} 2\pi y \, dy \quad (16)$$

For large values of  $x$ , when  $U_m, Y_m \ll 1$ ,

$$Q' = -q \frac{d}{dx} \left\{ \pi \rho_e r_e^2 u_e \frac{Y_{O_2 e}}{\nu} \frac{3}{7} \frac{\xi_f^{7/2}}{1 - \xi_f^{3/2}} \right\} \quad (17)$$

where

$$1 - \xi_f^{3/2} = \frac{Y_{O_2 e}/Y_m}{\nu + Y_{O_2 e}} \quad (18)$$

and  $r_e$  and  $Y_m$  are given, as functions of  $x$ , by Eqs. (11a) and (12).

### AERODYNAMIC EFFECTS OF HEAT RELEASE

If large perturbations in the flow field are to be avoided, heat addition rates per unit volume must be kept small. This is the case in diffusion flames more often than in premixed flames, resulting in an advantage of the diffusive mode over other modes of supersonic combustion.

In order to study, at least qualitatively, the influence of heat addition on the pressure field, we will linearize the conservation equations, as is done by B. T. Chu [13] when analyzing the pressure waves generated by heat addition. The following equations are obtained

$$u_e \frac{\partial \rho}{\partial x} + \rho_e \nabla \cdot \vec{v} = 0 \quad (19)$$

$$\rho_e u_e \frac{\partial \vec{v}}{\partial x} = -\nabla p \quad (20)$$

$$\rho_e u_e T_e \frac{\partial}{\partial x} c_v \ln(p/\rho^\gamma) = Q''' \quad (21)$$

Where  $Q'''$  is the heat addition rate per unit volume. Viscous effects are neglected.

From these relations we deduce

$$\nabla^2 p - M_e^2 \frac{\partial^2 p}{\partial x^2} = - \frac{u_e}{\gamma c_v T_e} \frac{\partial Q'''}{\partial x} \quad (22)$$

where  $M_e$  is the Mach number of the uniform air stream. Equation (22) must be solved with the appropriate boundary conditions corresponding to the combustion chamber walls.

Equation (22) indicates that heat addition is equivalent to the introduction of sources of strength

$$w''' = Q''' / \gamma c_v T_e \quad (23)$$

As is well known, a slender body of revolution in a uniform, unbounded, supersonic stream may be represented by a source distribution along the axis of strength  $w'$ ;

$$w' = \rho_e u_e \frac{dA}{dx} = \frac{Q'}{\gamma c_v T_e} \quad (24)$$

where  $A$  is the cross section of the body. Equation (24) indicates that, at least in an unbounded stream, the aerodynamic effects of heat released along a line in the flow direction, are identical to those introduced by a body of revolution with its axis on the heat-release line and with cross section.

$$A(x) = Q / \rho_e u_e \gamma c_v T_e \quad (25)$$

where  $Q(x)$  is the total heat release up to section  $x$ , as given in Eq. (16.)

Equation (25) may be used to study qualitatively the aerodynamic effects of heat addition in more complicated situations.

A large heat release in a given section gives rise to shock waves similar to those introduced by a blunt body. Also, by spreading the heat release in the flow direction, the equivalent body becomes slender and, therefore, the wave drag and corresponding pressure losses decrease.

In an unbounded uniform stream, Eq. (22) has the well-known solution:

$$\frac{p(x, y, z) - p_e}{\rho_e u_e^2} - \sqrt{M_e^2 - 1} \int_{-\infty}^x d\alpha \int_{-\infty}^{\beta} d\beta \int_0^x \frac{\partial Q'''(\alpha, \beta, \tau)}{\partial \tau \gamma c_v T_e \rho_e u_e} d\tau \quad (26)$$

$$\left[ \frac{(\tau - x)^2}{M_e^2 - 1} - (z - \alpha)^2 - (y - \beta)^2 \right]^{1/2}$$

The integration must be extended to those regions where the integrand is real (i.e., forward Mach cone).

## CHEMICAL NONEQUILIBRIUM EFFECTS

We aim in this section to show the effects of large, although finite, chemical reaction rates on the structure of diffusion flames. As an intro-

ductive example we will consider the study of one-step chemical reactions in which the forward reaction is dominant. The application to the hydrogen-air combustion will be considered in a subsequent paragraph.

It is possible to show [14, 15] that, for large reaction rates, the chemical reaction takes place only in a very thin region or "chemical boundary layer." Within the reaction zone, convection effects may be neglected compared with the much more important effects of chemical reaction, conduction and diffusion normal to the flame. The governing equations reduce in this region to ordinary differential equations. The kinetics of the reactions appears in the solution, but the temperature, being close to the adiabatic flame temperature, may be taken as constant for the determination of the reaction rates.

On the other hand, the solution with the assumption of infinite reaction rates (which we shall call the Burke-Schumann solution) represents the true solution outside of the reaction zone. It may also be used to calculate the flame position and fuel consumption per unit flame area.

At the boundaries of the reaction region both the chemical boundary layer and Burke-Schumann solution must coincide.

## GENERAL EQUATIONS

We shall begin by writing the species and energy conservation equations for the laminar flow of a mixture of three reacting species: fuel, oxidizer and products (subscripts 1, 2 and respectively). Any inert species will be considered as products. With the usual assumptions in the study of reacting flows, in particular equality of mass and thermal molecular diffusivities, we have

$$L(Y_i) = \frac{w_i}{\rho} \quad (27)$$

$$L\left(\frac{c_p T}{q}\right) = -\frac{w_1}{\rho} \quad (28)$$

where

$$L(\phi) = \bar{v} \cdot \nabla \phi - \frac{1}{\rho} \nabla \cdot (\rho D \nabla \phi)$$

and

$$\frac{w_1}{\rho} = \frac{w_2}{\nu \rho} = -\frac{w_3}{(1 + \nu) \rho} = -g(T, p) e^{-E/RT} Y_1^a Y_2^b \quad (29)$$



Equation (28), where viscous dissipation effects have been neglected, is only valid for low speed flows. For high-speed boundary-layer flows, if the Prandtl number is close to one, we must add the kinetic energy  $u^2/2q$  to the thermal energy  $c_p T/q$  for Eq. (28) to be valid.

Independently of the value of the reaction rates, from Eqs. (27) and (28) we obtain

$$L(Y_1 - Y_2/\nu) = 0 \quad (30)$$

$$L(Y_1 + c_p T/q) = 0 \quad (31)$$

Hence, for the "reduced" concentration  $(Y_1 - Y_2/\nu)$  and "reduced" temperature  $(T + q/c_p Y_1)$ , we have just a mixing problem independent of the reaction rates. For high-speed boundary-layer flows, we must use the total enthalpy  $(c_p T + q Y_1 + \frac{1}{2} u^2)$  instead of the "reduced" temperature.

If  $(Y_{1j}, 0, T_e)$  are the values of  $(Y_1, Y_2, T)$  far from the reaction region (at the boundary of the mixing region) on one side of the flame, while  $(0, Y_{2e}, T_e)$  are the corresponding values on the other side, we may easily deduce that

$$\frac{Y_{2e} + \nu Y_{1j} c_p (T - T_e)}{Y_{1j} Y_{2e} q} + \frac{Y_1}{Y_{1j}} + \frac{Y_2}{Y_{2e}} = 1 \quad (32)$$

is an integral of Eqs. (27) and (28) independently of the reaction rates and mixing process.

For later reference let us write the solution of Eqs. (30) and (31) in the form

$$Y_1 - Y_2/\nu = f_1(\bar{x}) \quad (33)$$

$$Y_1 + c_p T/q = f_2(\bar{x}) \quad (34)$$

Here, contrary to Eq. (32),  $f_1$  and  $f_2$  depend on the details of the mixing.

## CHEMICAL EQUILIBRIUM SOLUTION

The system of Eqs. (28) and (29), that must be solved together with the momentum equation, is extremely complicated, so that further approximations are desirable.

The left hand side of these equations is the inverse of a mechanical time  $t_m$ , of the order of the ratio of the overall characteristic length and velocity.

On the other hand, the factor  $g \exp(-E/RT)$  that appears in the right hand side, involving a frequency factor  $g$  and the Arrhenius factor, is the inverse of a chemical time  $t_c$ .

Whenever  $t_m/t_c \ll 1$ , the chemical production terms may be neglected, and "frozen flow" is obtained.

For large  $t_m/t_c$ ,  $w_1/\rho$  would be extremely large if  $Y_1^a Y_2^b$  would not take a small value. Hence, in the limit  $t_m/t_c \rightarrow \infty$ , we may write

$$Y_1^a Y_2^b = 0 \quad (Y_1 = 0 \text{ or } Y_2 = 0) \quad (35)$$

or, what amounts to the same,  $w_1/\rho = 0$ .

This last limiting process gives rise to the so-called "chemical equilibrium" approximation, which for one-step chemical reactions may be termed the Burke-Schumann approximation.

Taking Eq. (35) into Eqs. (33) and (34), we obtain

$$Y_2 = 0; \quad Y_1 = f_1(\vec{x}); \quad c_p T/q = f_2(\vec{x}) - f_1(\vec{x}) \quad (36a)$$

on the fuel side of the flame, and

$$Y_1 = 0; \quad Y_2 = -\nu f_1(\vec{x}); \quad c_p T/q = f_2(\vec{x}) \quad (36b)$$

on the oxidizer side of the flame. The flame surface, determined by  $Y_1 = Y_2 = 0$ , is given by

$$f_1(\vec{x}) = 0 \quad (37)$$

The flame temperature, from Eq. (32), is given by

$$T_f = T_e + \frac{q}{c_p} \frac{Y_{1j} Y_{2e}}{Y_{2e} + \nu Y_{1j}} \quad (38)$$

## NONEQUILIBRIUM EFFECTS

The Burke-Schumann equilibrium, "thin flame" solution, Eq. (36), obtained by taking the ordinary limiting process  $t_m/t_c \rightarrow \infty$ , identically satisfies the complete system of Eqs. (27) and (28) and their boundary conditions. It is not the uniformly valid solution, because concentration and temperature derivatives normal to the flame surface are discontinuous at the flame. These discontinuities are caused by the fact that, in the limiting process, the nonequilibrium reaction region shrinks to a surface.

It is possible, for large reaction rates, to analyze the structure of this reaction zone by using boundary-layer techniques [14, 15]. In order to do this, we first write, in nondimensional form, the conservation equations in boundary-layer coordinates parallel and normal to the infinite rate surface. The fact that second derivatives normal to the flame are infinite in the Burke-Schumann solution indicates that they play a dominant role in the large, but finite, reaction rate case. Hence, we stretch the coordinate

normal to the flame, so that the diffusion and chemical production terms are not lost in the limiting process  $t_m/t_c \rightarrow \infty$ .

Then, within the reaction zone, the conservation Eqs. (27) and (28) reduce to

$$\frac{1}{\rho} \frac{\partial}{\partial y} \left( \rho D \frac{\partial Y_i}{\partial y} \right) = - \frac{w_i}{\rho} \quad (39)$$

$$\frac{1}{\rho} \frac{\partial}{\partial y} \left( \rho D \frac{\partial T}{\partial y} \right) = \frac{q}{c_p} \frac{w_1}{\rho} \quad (40)$$

In these equations the terms due to convection and diffusion in the direction parallel to the flame have disappeared.

To obtain the boundary conditions, we require the outer limit ( $y \rightarrow \infty$ ) of the solution of Eqs. (39) and (40) to coincide with the inner limit of the Burke-Schumann solution.

The solution thus obtained of Eqs. (39) and (40), valid within the reaction zone, together with the equilibrium solution, valid outside of the reaction region, represent the first approximation for the entire flow field. Figure 8 schematically shows the corresponding temperature and mass fraction distributions.

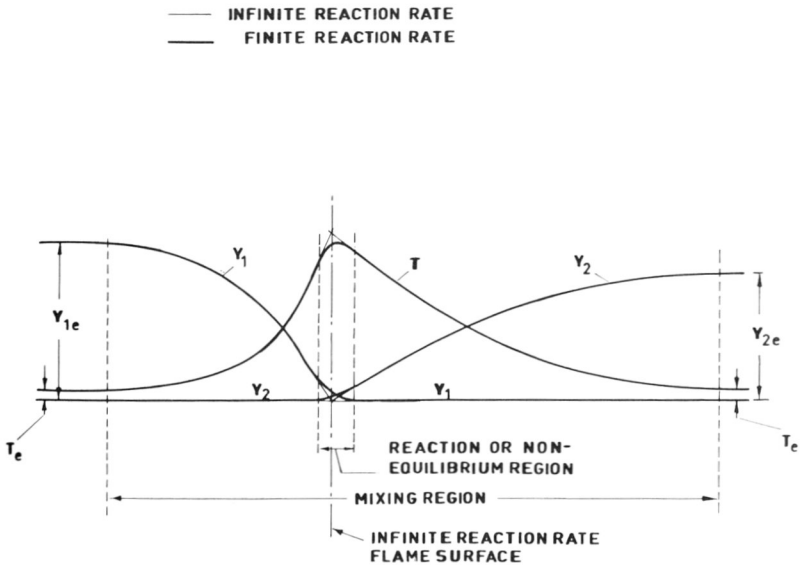


Figure 8.

To obtain further approximations, the “inner and outer expansions” procedure must be used. This has been done by Fendell [16] for the combustion of initially unmixed gases in inviscid axially symmetric stagnation point flow.

Notice that in Eqs. (39) and (40) only derivatives with respect to  $y$  appear; hence, they may be solved as ordinary differential equations in which  $x$  acts as a parameter.

In addition, the temperature within the reaction zone remains close to the adiabatic, thin flame, temperature  $T_f$ . This fact may be used in order to simplify the determination of the mass production terms.

Deviations, of the temperature in particular, from the equilibrium values may be shown [15] to depend on the ratio  $\rho_f^2 D_f / \dot{m}^2 t_c$ , where  $\dot{m}$  is the mass consumption rate of fuel per unit flame area. The reaction-layer-to-mixing-layer thickness ratio depends on the same parameter.

Figure 9 shows the temperature distribution along the infinite reaction rate flame surface as calculated in Ref. 14 for the combustion of two parallel streams of fuel and oxidizer. Radiation from the flame will have an even stronger dependence on  $x$ ; and a fairly sharply defined ignition length becomes apparent. This length may be estimated by using the “simple” chemical boundary-layer solution.

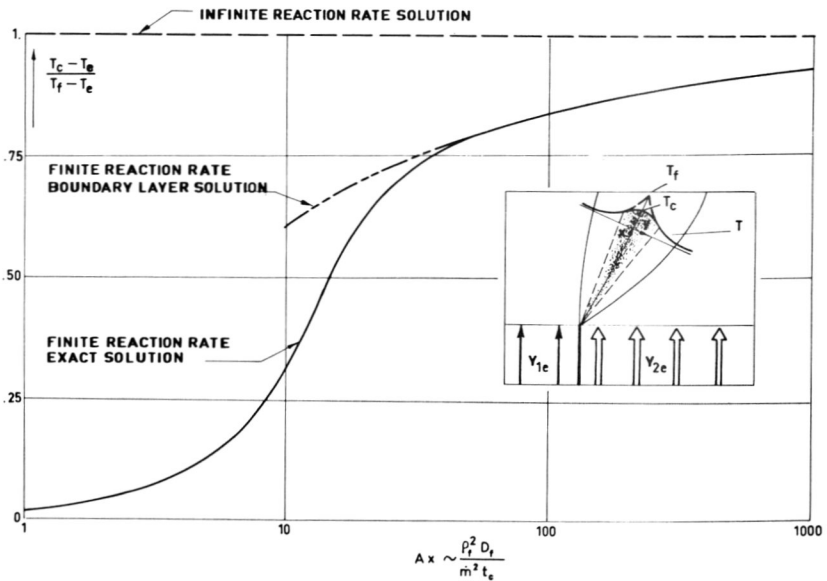


Figure 9.

## EXTENSION TO TURBULENT FLOWS

No reliable information exists on the values of the reaction rates on turbulent flows.

If following a generalized practice in the study of turbulent reacting flows, we use the values of the laminar-flow reaction-rate expressions in which we insert the turbulent mean temperature and concentrations, we may easily extend the foregoing results to the turbulent case. In particular, within the reaction zone, Eqs. (39) and (40) apply, if  $D$  is substituted by the turbulent diffusivity at the flame  $\epsilon$ .

## HYDROGEN-AIR COMBUSTION

Owing to its high calorific value, its high capacity for cooling, its thermal stability, and its rapid reaction rates, hydrogen is the most promising fuel for a hypersonic engine. The characteristics of such common liquid fuels as kerosene are also of interest in the range of Mach numbers from 4 to 25, but the specific impulses attained with kerosene are, at most, 40 percent of the corresponding values for hydrogen in the same Mach number range.

In the case of the hydrogen-air reaction, which in the range of temperatures of interest in supersonic combustion (1000°K–2500°K), reduces to hydrogen-oxygen reaction with nitrogen as a diluent, the kinetic scheme proposed by Schott [17] for the determination of the reaction rates is widely used. Almost nothing is known on the influence of turbulence on the reaction rates. In any case, no overall reaction rate expression exists for the hydrogen-oxygen reaction.

The main difficulty in the application of the results to the study of the internal structure of the hydrogen-oxygen flame is related with the fact that eight simultaneous chemical reactions and six different species are significant in the range of temperatures 1000°K–2500°K. As a result, no reliable overall reaction rate expression exists.

From the consideration of more simple chemical kinetic schema, possessing some features of the complete scheme, we infer the possibility of studying the structure of the reaction zone, by dividing it into several layers. In each of these layers some of the reactions are dominant, and an overall reaction rate expression may exist. Work is under way on this direction, and results will be published elsewhere.

## MAIN NOMENCLATURE

- $a$  = radius of the injector
- $b$  = width of the boundary layer of the initial mixing region
- $c$  = experimental growth coefficient in Eq. (4)

$D$	=	diffusion coefficient
$m$	=	velocity ratio of the jet = $u_e/u_j$
$p$	=	static pressure
$q$	=	heat release per unit mass of fuel
$r$	=	radius of the jet
$T$	=	temperature
$u$	=	velocity component in the $x$ direction
$\bar{u}$	=	characteristic mean velocity in Eq. (4)
$U$	=	dimensionless velocity = $(u - u_e)/(u_j - u_e)$
$v$	=	velocity component in the $y$ direction
$x$	=	axial coordinate
$y$	=	radial coordinate
$Y$	=	reduced mass fraction = $\frac{\nu Y_{H_2} - Y_{O_2} + Y_{O_2e}}{\nu + Y_{O_2e}}$
$Y_i$	=	mass fraction of species $i$
$\gamma$	=	$c_p/c_v$ ratio of specific heats
$\epsilon$	=	turbulent diffusivity
$\rho$	=	density of the mixture
$\nu$	=	stoichiometric ratio of oxidizer to fuel

## SUBSCRIPTS

$c$	=	indicates compressible conditions
$e$	=	values external to the mixing region
$f$	=	values at flame sheet
$i$	=	indicates incompressible conditions
$j$	=	values in the inviscid core of the jet
$m$	=	values in the axis of main region of the jet
$n$	=	values at the end of the transition zone of the jet
$H_2, O_2$	=	indicates chemical species

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## COMMENTARY

CHARLES A. LINDLEY (*The Aerospace Corporation, El Segundo, Calif.*): The authors are to be congratulated on their analysis of a very difficult subject. While the difficulties and uncertainties in extension of the analysis to the turbulent case, of which they speak, are very real, it is necessary that we come to understand this case and the effect, if any, of combustion on the turbulent mixing rates. From the practical engine point of view, it is extremely important that the mixing and reaction be made to occur in the minimum possible length; otherwise weight and heat-transfer considerations will rule out the supersonic combustion ramjet type for many applications.

In any complicated field of fluid mechanics such as this, it is necessary that analysis and well-planned experiment go hand in hand. Is an appropriate experimental program under way in another portion of your organization, or in any other place available to you, and with whom you can interact?

**REPLY**

First we want to thank you for your kind words about our work.

Of course, we agree on the necessity of well-planned experiments parallel to the theoretical investigation, especially in a new field like supersonic combustion. Unfortunately, we have not had, up to now, the opportunity to set up an experimental program in this field. However, a rapidly growing number of experimental results, published elsewhere, is now being made available.